

Fatigue Crack Growth Predictions in Aging Aircraft Panels Using Optimization Neural Network

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An optimization-based neural network method is developed to predict fatigue crack growth and fatigue life of multiple site damage panels found in aging aircraft. The method utilizes an optimization solution to predict the probable crack path based upon the initial panel configuration and accounts for lead crack spanning, small multiple site damage, and plasticity zones. The approach of the neural network was motivated by the optimization analysis and the time-consuming computational analyses for multiple site damage problems. The present neural network method was able to predict crack propagation and fatigue life and compares well with the experimental data of fatigue tests on the 2024-T3 aluminum panels.

I. Introduction

MULTIPLE site damage (MSD) refers to multiple cracks of arbitrary length that commonly occur along rows of fastener holes in the fuselages and wings of commercial and military aircraft. Such cracks are extremely difficult to detect with a variety of existing nondestructive evaluation techniques. The difficulty in detecting MSD is caused in part by the fact that cracks may occur at holes along splices, may link up to form bigger cracks, and are usually not visible to the eye or are not detected during scheduled checks. Further, because U.S. commercial aircraft are built to withstand longitudinal cracks in the fuselage that span two adjacent bays, it is assumed that cracks can be routinely detected before they reach this critical crack length. Recently, considerable efforts have been devoted to studying the widespread fatigue damage behavior and its effects on structural integrity as it greatly reduces the fatigue life of aircraft panels with MSD.

Material degradation as a result of widespread fatigue damage is quantified in terms of reduction in strength, fracture toughness, and fatigue life. Several approaches have been used to estimate the residual strength of aircraft panels with multisite damage. Swift¹ developed an analytical method to determine the residual strength of a panel based on yield stress method but taking into account the plasticity and crack interaction effects. Tong² derived a simple formula to predict the reduction of the residual strength due to rivet holes in aircraft panels. Several techniques have also been proposed for establishing multisite fatigue damage stress intensity factors, for example, finite element or boundary element methods and superconvergent and compounding methods,³⁻¹² which are described briefly in Sec. III.

The current stress analysis and fracture mechanics methods for material degradation and fatigue crack growth prediction are difficult to implement in complex aircraft structures with MSD. Also, significant MSD cracking may develop before it can be reliably detected by many nondestructive evaluation (NDE) techniques. The approach of neural network (NN) methods was motivated by the optimization analysis and the time-consuming computational analyses for MSD problems. When solving MSD problems using computational methods, many assumptions have to be made. However, models based on neural networks may be efficient in those situations where experimental data exist, and those models do not have to make many assumptions to obtain prediction. Several authors used NN methods for structural damage detection,¹³⁻¹⁶ fracture

problems,¹⁷ optimization problems,^{18,19} and structural reliability.²⁰ Based on the existing literature, it appears that there is a need to explore the approach of NN methods for predicting widespread fatigue damage in aging aircraft because such methods can provide real-time performance, can be efficient, and can complement the experiments and the analytical and computational methods.

The objective of this paper is to develop an optimization NN model to predict the fatigue life and fatigue crack growth for panels with MSD. Two NN models are combined in an innovative way to predict the fatigue crack growth of MSD panels. The local crack growth behavior around the holes is predicted using backpropagation network, whereas the global behavior of the panel is predicted using an optimization NN model. The present optimization NN model is demonstrated by comparing the predictions with available experimental data and analytical methods for selected examples on 2024-T3 aluminum panels with MSD. The NN approach will serve as an alternative and will complement the existing analytical and computational methods.

II. Stages of Multiple Site Damage

Multiple site damage refers to the existence of simultaneous fatigue cracks in aircraft panels. This damage occurs along rows of fastener holes in the fuselage and wings of aging aircraft. Three different stages have been identified in aircraft panels with MSD.⁵ These are described next.

1) Stage I (local stage): When cracks are too small to affect neighboring cracks, then we define the crack to be in the local stage. The local stage can be divided into two phases: a) the crack initiation phase and b) the continuation of local growth until cracks obtain the transition crack length where they influence other cracks within the panel. The crack growth at this stage is fairly linear. Figure 1a shows how the crack propagates for a two-hole panel. Simulation of crack propagation in stage I should be the most direct and quickest to implement. The difficulty is to adequately estimate when the distance between adjacent holes is close enough to allow for interaction. This is, of course, based upon the material used and the intensity and the location of stress (along with other factors). However, for the aluminum panels of the test data, the estimation of transition crack length can be calculated mathematically.⁵

2) Stage II (MSD stage): The crack is no longer constrained in its own separate universe as another crack has begun to influence crack propagation. We shall see that the MSD stage of crack growth exponentially increases crack growth rates. What must be considered is that an outside influence is affecting local growth rates and that the local crack must now be affecting the growth pattern of adjacent cracks. The area between the two cracks that are influencing each other is called a plasticity zone. This area, as shown in Fig. 1b, is as important to consider as the configuration of the actual cracks.

3) Stage III (postlinkup stage): Cracks between adjacent holes have linked up to form one continuous crack. This leads to rapid

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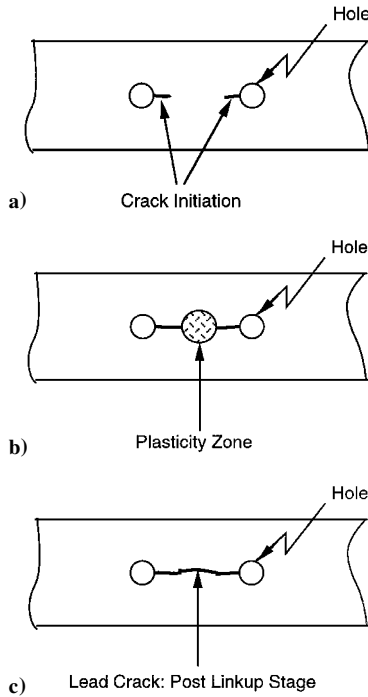


Fig. 1 MSD panels showing crack initialization and the plasticity zone and crack growth showing the postlinkup stage.

panel failure because lead cracks greatly influence the adjacent holes (cracks) (see Fig. 1c). Holes with adjacent lead cracks should be evaluated differently from holes with MSD cracks. The question arises whether the postlinkup stage can be assimilated into the MSD stage of crack growth. Because the crack length spans multiple holes, the plasticity zone's area will have increased drastically.

III. Current Approaches to MSD Prediction

Several techniques have been proposed for establishing MSD stress intensity factors, for example, finite element or boundary element methods and superconvergent and compounding methods. These methods are briefly described next.

A. Finite Element Alternating Method

In this method, first a coarse finite element or boundary element mesh is used to obtain the stresses in the uncracked body but ignores the detailed stress state near the crack tip. The effects of cracks are then assessed by erasing the tractions at the crack locations. This is accomplished by replacing a finite cracked body with an infinite one (because it has an analytical solution) and iteratively solving until the traction conditions on the boundaries are satisfied on the finite body. The stress intensity factors are obtained from the converged solution.³ Different types of MSD specimens were analyzed by various authors. Dawicke and Newman⁴ used boundary elements to analyze the stresses for uniaxially loaded flat panels with MSD. Their numerical results are within 20% of the experimental lives for different crack configurations studied. Atluri and Tong,³ Park et al.,⁵ Park and Atluri,⁶ and Pyo et al.⁷ developed and extensively applied this finite element alternating method (FEAM) to a variety of problems in aging aircraft and structures and found good agreement with other alternate solutions. Park et al.⁵ used FEAM to investigate the effects of cracks with and without composite-patch repairs in MSD problems. A simple and efficient computational method was developed by Park and Atluri⁶ to study the fatigue crack growth of MSD cracks. Recently, Pyo et al.⁷ extended the FEAM into the elastic-plastic regime and studied the MSD linkup phenomenon using T^* fracture criterion.

B. Finite Element Superconvergent Methods

The MSD stress intensity factors at the crack tip are obtained using p -version finite element analysis by Actis and Szabo.⁸ This method involves solving an MSD configuration with finite element analysis and extracting the stress intensity factors from a contour integral

on an arbitrary circular path around the crack tips. They did not verify their stress intensity factor results with any experimental data; however, their approach was demonstrated with respect to efficiency and reliable control of numerical errors. They also demonstrated, using their techniques, that coarse meshes could be used for MSD crack tip solutions.

C. Compounding Methods

Partl and Schijve⁹ and Nishimura et al.¹⁰ used compounding methods to solve for MSD stress intensity factors. Aluminum panels having a collinear row of open holes with three center holes cracked were analyzed by Partl and Schijve.⁹ Their results demonstrate that simple compounding techniques produce acceptable results for MSD problems. Also, their predicted results agreed to within 14% of experimental data. Using different correction factors along with the compounding methods, Nishimura et al.¹⁰ analyzed MSD problems with many holes. Their predictions of fatigue lives agreed to within 17% with the experimental data for all the tested specimens. Recently, Moukawsher¹¹ studied the fatigue life of MSD panels using linear elastic fracture mechanics and compounding methods. Their fatigue life predictions were in general good agreement with the experimental data for most cases when the crack tip interaction was included in their analytical model. Based on the existing literature and as discussed earlier, it would be of interest to explore the approach of inverse methods such as NN for fatigue crack growth predictions of MSD panels because of the large variability that exists in crack data.

IV. Neural Network Model

The objective of the optimization NN model is, given an MSD panel, to determine 1) how fast each crack propagates at the fasteners and 2) when the panel experiences fatigue failure based on a critical crack size in the material. Figure 2 shows the overview of the proposed approach to predict the fatigue life of MSD panels. Two NN models are combined in an innovative way to predict the fatigue crack growth of MSD panels. The local crack growth behavior around the holes is predicted using a backpropagation network, whereas the global behavior of the panel is predicted using an optimization NN model. A multiple site damage prediction neural network (MSDPNN) was developed based on optimization networks similar to a Hopfield²¹ and Hopfield and Tank²² network. However, in the MSDPNN method, each neuron in the network contains a finite amount of energy, and exactly how much energy a neuron contains is directly proportional to the effect the neuron has on the network. The primary goal of each neuron in the network is to accumulate or to dissipate energy. In the MSDPNN, both the neural network and the data will reside within the network. Each of the neurons will contain an energy E and the pertinent data about the crack. Therefore, each neuron consists of the following information: 1) γ , the sums of neural energy; 2) LCL, the left crack length; 3) LGL, the left gap length; 4) RCL, the right crack length; and 5) RGL, the right gap length. The total system energy is defined as $T = 1$. Therefore, the initial energy assigned to this neuron and each neuron in the system is $1/N$, where N is the number of neurons. The maximum energy any neuron can obtain is therefore $\sqrt{N/N}$. At this point, this neuron would have become an optimum member of the final panel configuration and therefore an optimum solution.

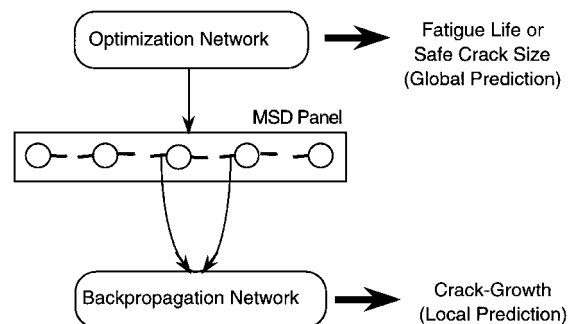


Fig. 2 Global/local view of the crack growth prediction model.

As the lead crack is completed, the energy is maximized based upon a translated Sigmoid function²³ given by

$$F(u) = \begin{cases} \frac{-\sigma}{1 + \exp(-\alpha u)} + t & \text{if } u \neq 0 \\ +\rho & \text{if } u = 0 \end{cases} \quad (1)$$

where σ is a scaling term, u is an output of the neuron, and α is a constant. Consequently, every neuron in the system starts off at 0 on the function scale mapped to $1/N^2$ neurons. When convergence begins, energy transfer is very fast (relative to later stages), and neurons will quickly begin positioning themselves. Thus, in later stages neurons that have found themselves in optimal positions (high up on the function scale) will slowly begin interacting among themselves to determine a final optimal state. The parameters within the function can be used to modify tangency in early and/or late stages of convergence. This is a strictly increasing logistic function that exhibits smoothness and asymptotic properties except at the discontinuity, which would allow a near-continuous graded output response as a function of inputs instead of an on/off model as in the classical Hopfield model. The operational features of the MSDPNN model can be summarized as follows.

1) Use a modified formal neuron of McCulloch and Pitts (see Ref. 22).

2) Implement a modified Sigmoid function for convergence purposes.

3) The state of the network is to be defined as

$$S = [s_1, s_2, \dots, s_N]^T$$

where the system is composed of N neurons.

4) Time $T \rightarrow \infty$ to allow for a stable state to exist. The system converges to a stable state when the neurons representing the holes no longer absorb or release any energy over a period of time.

5) Output from a neuron is feedback to all other neurons.

6) No self-feedback exists in the network.

7) Nonsynchronous updating of neurons in the system is used. (This is directly related to what occurs in biological systems.)

8) The optimization problem is solved when the network reaches a stable state (pathwise energy consolidation).

The learning process involves the Perceptron[®] learning algorithm 25 and now consists of three key stages: stage 1, the neural network is stimulated by the environment; stage 2, the MSDPNN undergoes change due to this stimulation; and stage 3, the MSDPNN responds in new ways to the environment because changes have occurred within the NN's internal structure.

A. Energy Function

Let N be a set of neurons (for a given panel) that represents the N holes on a panel. An entire panel can be represented as a matrix U of size N^2 neurons. In matrix U , the outputs will be labeled as U_{ni} where n refers to the hole number and i refers to the position of failure. The energy function requirements correspond to a stable state, yielding a stationary value for the energy by minimization or optimization. The energy requirements can be stated as follows.

1) Energy minima must favor states that have holes appearing once on the panel. The first term of the energy function, T , is zero if and only if no more than one element in each row of the output unit matrix is 1. This can be satisfied by a term in the energy function T ,

$$-A\delta uv(1 - \delta ij)$$

where $\delta uv = 1$ if $u = v$ and $\delta uv = 0$ if $u \neq v$. The first δ is zero except on a single row where $X = Y$. The quantity in parentheses is 1 unless $i = j$. This ensures that a unit inhibits all units on its row but does not inhibit itself (lateral inhibition).

2) Energy minima must favor states that include all N holes. The contribution of this term in the energy equation involves a sum of all of the outputs. Thus, we include a global inhibition B such that each unit in the network is inhibited by this constant amount where B is normalized between 0 and 1.

3) Energy minima must favor states in which total fatigue failure occurs. The contribution of the third term in the energy equation [see Eq. (2)] is a universal constant C that ensures movement toward fatigue failure of the entire panel.

4) Energy minima must favor states in which plasticity causes rapid movement toward failure. This constraint is satisfied by term D that has information about the distance between crack ends for each hole. The inputs from the $n - 1$ and $n + 1$ relative hole positions on the panel may directly affect hole n when plasticity becomes an issue. Units on adjacent columns represent holes that might come either before or after the hole on column j . The term D ensures that the units representing holes farther apart will receive the largest inhibitory signal. Plasticity should not affect holes that are far apart.

5) Energy minima must favor states in which crack growth occurs at the appropriate rate even before plasticity accelerates growth. This constraint ensures that the growth is stable and that no external crack influences local crack growth until plasticity causes rapid crack growth. This requirement is denoted by E in the energy equation.

6) Energy minima must favor states in which an initial panel state existed. For each hole a function must be included in the constraint that accurately weights the rows to reflect the initial panel configuration. Therefore, crack growth occurs at the appropriate rate and distance for the initial configuration. This requirement is satisfied by term F in the energy equation. This constraint allows an image of the initial panel to be imposed onto the neural network.

7) Energy minima must favor states in which a failure constraint either causes convergence or sets an exclusive flag to mark the point of panel failure. This requirement is satisfied by term G and would allow for convergence or termination for panel failure other than the trivial case of total crack propagation.

Now the entire connection matrix can be defined as

$$T_{xi,yj} = -A\delta xy(1 - \delta ij) - B - C - \frac{(D + E + F + G)}{\beta} \quad (2)$$

The term $1/\beta$ weights the values of the last four constraints so that appropriate growth rates are ensured. In the present implementation, a backpropagation (BP) network^{24,25} is used to solve constraints 4–7 collectively to obtain the values for D , E , F , and G . During the optimization process in the Hopfield network, the parameters A , B , and C are adjusted dynamically to achieve a stable energy state. The position of failure for each hole can be determined by observing the convergence matrix U . The rate of convergence can be examined by saving the intermediate U . The number of cell activations are also tracked during the iteration process. This value divided by the size of the network would give an insight into the cycles at each snapshot of the matrix including the matrix for the failed panel.

V. Results and Discussion

A computer program was written in C language based on the NN model described in the preceding section and implemented on Sun workstations. The MSDPNN method simulates path propagation for arbitrary MSD panels until fatigue failure occurs. In the simulation, fatigue failure corresponds to a critical crack length, or the number of cycles exceeds a preset threshold. The objective of the simulation is, given an MSD panel, to determine 1) how fast each crack propagates at the fasteners and 2) how many cycles have elapsed before the panel experiences fatigue failure based on the fracture toughness of the material. Computer simulations have been run to predict the crack growth rates for two initial panels where the actual results are known from empirical data gathered by measured crack propagation of MSD aircraft panels under constant stress. The primary objective was to compare the correctness of the model against actual known test data¹¹ that was measured. The model has been tested on various MSD panels. Results from two of the test runs are given to illustrate the prediction capabilities of the proposed optimization approach for fatigue crack growth of MSD panels.

A. Experiment 1: Panel A

Figure 3 shows the initial configuration for this panel, and Table 1 provides the details for this panel.

The experiment was ran for 50,000 iterations, and at 37,890 cycles the panel experienced fatigue failure when the average crack length reached 0.2450 in. As the energy in the system converges to a steady state (defined as all energy exists in N neurons), neurons along each row begin either absorbing energy or releasing energy. A snapshot

Table 1 Crack and hole details of panel A used in experiment 1

Hole no.	Left crack	Right crack
1	0.046	0.042
2	0.042	0.062
3	0.066	0.079
4	0.065	0.083
5	0.114	0.085
6	0.067	0.068
7	0.082	0.079
8	0.071	0.062
Number of holes: 8		
Avg. hole diameter: 0.150		
Hole spacing (in.): 1.0		
Lead crack length: none		
Stress range: 10.69 ksi		

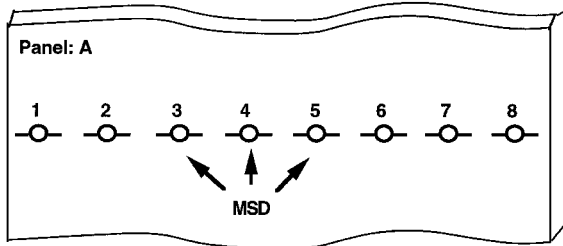


Fig. 3 Initial panel configuration for panel A used in experiment 1; note that each hole has MSD property.

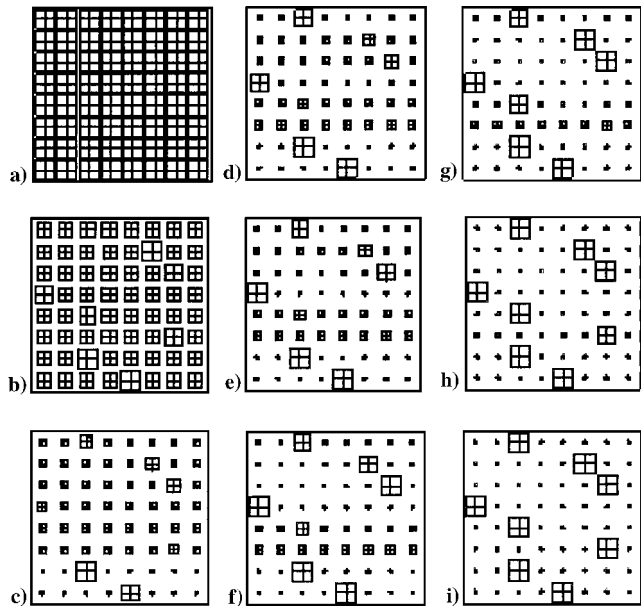


Fig. 4 Hinton-style energy plot showing the convergence for experiment 1; note that subplots a-i correspond to intermediate snapshots showing the current status of neurons.

of the energy distribution of the network is displayed at every 5000 iterations as a Hinton diagram in Fig. 4.

Figure 4a shows the initial energy configuration of the 64 neurons (8 holes \times 8 positions of failure for each hole), where all weights in the system are evenly distributed across the network, each with an initial energy value of $0.015625 (\frac{1}{64})$. The following parameter values were used for this simulation run: $\sigma = 2$, $\alpha = 4$, and $\beta = 3$. These values were determined empirically. Figure 4b displays the network at 5000 iterations, where convergence was beginning to favor certain neurons in the network. By 15,000 iterations, Fig. 4c, the energy has begun to converge to an optimum state defining which neurons in the system contain the best data of the final panel configuration. At 37,890 iterations, the network energy has converged to 8 individual neurons defining a steady state in the system as seen in Fig. 4i. At this point, we can determine that the optimal neurons in the network are $N_{0,2}$, $N_{1,5}$, $N_{2,6}$, $N_{3,0}$, $N_{4,2}$, $N_{5,6}$, $N_{6,2}$, and $N_{7,4}$. We evaluate the crack information in these 8 neurons at each 1000th iteration of the

Table 2 Crack and hole details of panel B used in experiment 2

Hole no.	Left crack	Right crack
1	0.140	0.150
2	0.104	0.103
3	0.108	0.069
4	0.067	0.421
5	0.421	0.064
6	0.050	0.053
7	0.064	0.076
8	0.000	0.000
Number of holes: 8		
Avg. hole diameter: 0.160		
Hole spacing (in.): 1.0		
Lead crack length: 1.29		
Stress range: 5.95 ksi		

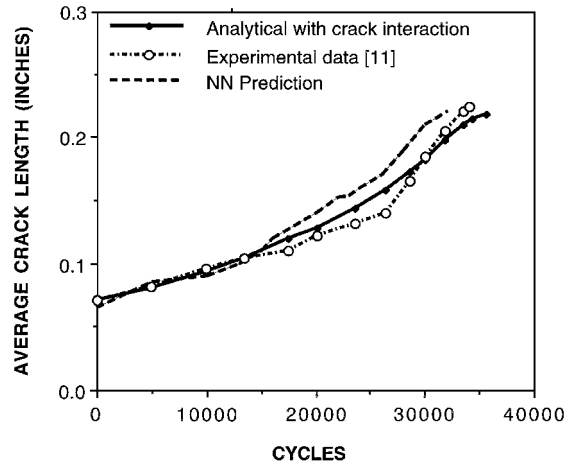


Fig. 5 Plot showing the comparison between MSDPNN prediction vs actual data¹¹ for experiment 1.

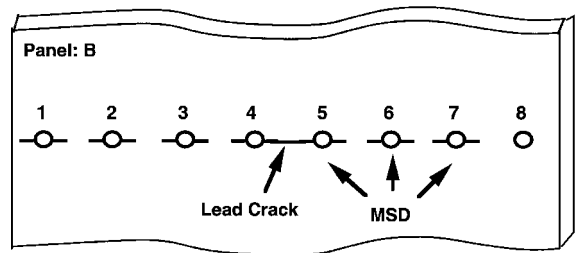


Fig. 6 Initial panel configuration for panel B used in experiment 2; note the holes with lead crack property and the eighth hole with no crack.

network to determine an optimal final panel configuration. This is done by plotting an average of the crack length data that are stored in each of the 8 converged neurons at each 1000th iteration. The information contained in the remaining neurons is discarded.

In Fig. 5, the solid line shown plots crack propagation of MSD panel A based on analytical methods¹¹ with crack interaction. The line with circles shows measurements of constant stress applied to a panel with the given initial configuration. The dashed line plots the MSDPNN prediction based upon the same initial panel configuration. The primary variance between the actual propagation and the predicted growth occurs between cycles 15,000 and 29,000 where the MSDPNN model predicted slightly higher growth rates than the test data revealed. This is due to the fact that the BP network, which provides growth rates for individual cycles, was trained with initial data containing high growth rates. However, the MSDPNN network was able to adjust to an optimum solution and, thereby, determine a final panel state that closely matched the final configuration of the test panel.

B. Experiment 2: Panel B

Figure 6 shows the initial configuration for this panel, and Table 2 provides the details for this panel. The following parameter values were used for this simulation run: $\sigma = 2$, $\alpha = 4$, and $\beta = 2$.

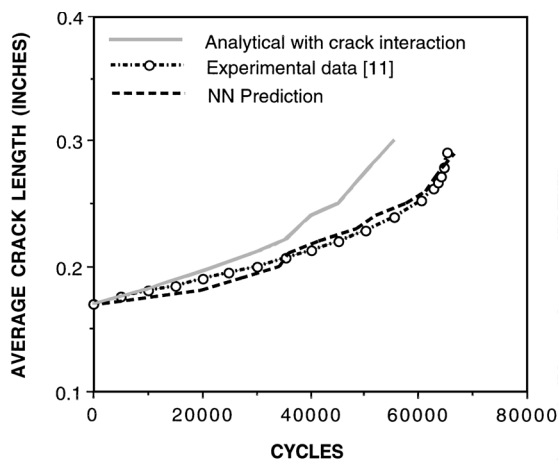


Fig. 7 Plot showing the comparison between MSDPNN prediction vs actual data¹¹ for experiment 2.

The experiment was run for 100,000 iterations, and at 68,910 cycles the panel experienced fatigue failure when the average crack length reached 0.3180 in. By 18,000 iterations the energy began to converge to an optimum state for neurons that corresponded to holes 1, 2, 3, 7, and 8. At 68,910 iterations the network energy converged to eight individual neurons defining a steady state in the system. We evaluate the information in these eight neurons to determine an optimum final panel configuration. In Fig. 7, the solid line shown plots crack propagation of MSD panel A based on analytical methods¹¹ with crack interaction. The line with circles shows measurements of constant stress applied to a panel with the given initial configuration. The dashed line plots the MSDPNN prediction based on the same initial panel configuration. The primary variance between the actual propagation and the predicted growth occurs at 62,000 cycles, where the MSDPNN model predicted slightly slower growth rates than the test data revealed. The MSDPNN network was able to adjust to an optimal solution and to determine a final panel state that matched average crack length within 4% of the final configuration of the test panel.

VI. Conclusions

Undetected MSD can significantly impair the structural integrity of aircraft. An optimization-based neural network method is developed to predict the crack growth as well as the fatigue life of panels with MSD for a given initial panel configuration. In the optimization neural network, each neuron corresponds to a hole, and it contains pertinent information relevant to existing crack conditions. As the crack grows, the neurons gain energy. A set of energy functions has been developed that governs how the neurons gain energy as the system begins to converge for an optimum solution. Two sets of experiments were conducted to evaluate the performance of the MSDPNN system. The results obtained from MSDPNN were compared with actual fatigue test data on aluminum panels. These results show that the model was able to predict the crack growth and panel failure fairly accurately with that of the experimental data. Preliminary results indicate that the proposed method is a promising approach for predicting crack growth for panels with MSD in aging aircraft.

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